Eur. Phys. J. C 4, 185–191 (1998)
DOI 10.1007/s100529800890

THE EUROPEAN
PHYSICAL JOURNAL C

QCD corrections to the forward-backward asymmetries of c and b quarks at the Z pole

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Received: 26 February 1998 / Published online: 2 June 1998

Abstract. Measurements of the forward-backward production asymmetry of heavy quarks in Z decays provide a precise determination of $\sin^2\theta^{\rm lept}_{\rm W,eff}$. The asymmetries are sensitive to QCD effects, in particular hard gluon radiation. In this paper QCD corrections for $A^{b\bar{b}}_{\rm FB}$ and $A^{c\bar{c}}_{\rm FB}$ are discussed. The interplay between the experimental techniques used to measure the asymmetries and the QCD effects is investigated using simulated events. A procedure to estimate the correction needed for experimental measurements is proposed, and some specific examples are given.

1 Introduction

Over the past few years experiments at LEP have made increasingly accurate measurements of the forward-backward asymmetries of Z decays to heavy quarks [1] leading to very precise determinations of the electroweak mixing angle. In particular the b asymmetry provides a determination of $\sin^2\theta^{\rm lept}_{\rm W,eff}$ with one of the smallest errors so far. Combined with the top mass measured at the TEVATRON, the precision reached in the measurement of $\sin^2\theta^{\rm lept}_{\rm W,eff}$ implies interesting constraints on the Higgs mass [1].

QCD corrections play an important role when interpreting the measured asymmetries in terms of $\sin^2\theta^{\rm lept}_{\rm W,eff}$. They mainly arise from hard gluon emission, which distorts the angular distribution of partons, compared with the pure electroweak process. The size of the effect may be calculated using perturbative QCD [2–5].

In experimental measurements, the sensitivity of the analysis to gluon radiation depends on the technique used, implying that corrections calculated in perturbative QCD are not directly applicable, but should be modified according to the details of the analysis. Until now, the experimental bias to the QCD corrections was considered only as an additional source of systematic error [6]. With the full statistics of four million hadronic Z decays collected by each LEP experiment this approach is no longer satisfactory, leading to a non negligible contribution to the the total error on $\sin^2\theta^{\rm lept}_{\rm W,eff}$.

In this paper a procedure is proposed to evaluate the correction to be applied to any given analysis. Analytical calculations of the QCD corrections to the heavy quark asymmetries are reviewed and applied to the experimental case. A method is proposed to combine these theoretical calculations with experimental effects estimated using Monte Carlo models. The sources of experimental bias are discussed in some detail. Finally some examples of experimental biases calculated for existing LEP measurements are presented.

2 QCD corrections

In the Standard Model, the differential cross-section for the process $e^+e^- \to f^+f^-$ in its most general form is given by [3]

$$\frac{{\rm d}\sigma}{{\rm d}\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,\sigma_{\rm U} + \frac{3}{4}\sin^2\theta\,\sigma_{\rm L} + \frac{3}{4}\cos\theta\,\sigma_{\rm F}\ , \quad (1)$$

where $\sigma_{\rm U,L}$ are the unpolarised and longitudinally polarised cross-sections and $\sigma_{\rm F}$ is the difference between the right- and left-handed polarised cross-sections. This cross-section describes the decay of a spin one boson, with the angle θ measured between the incoming and the outgoing fermions. The angular decomposition is still valid if radiative effects are included or if the thrust axis oriented according to the direction of the outgoing fermion is used.

At the Born level if the fermions were massless no longitudinal component would be present, while for massive fermions a small longitudinal component is expected. In the presence of gluon radiation in the final state, the relative contributions of the different cross-sections are modified compared to the lowest order; in particular, the longitudinal cross-section increases and is no longer negligible. Equation 1 can be rewritten as

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{6+2a}(1 + a\cos^2\theta) + A_{\mathrm{FB}}^{q\bar{q}}\cos\theta \ , \tag{2}$$

where the forward-backward asymmetry $A_{\mathrm{FB}}^{q\bar{q}}$ and the shape coefficient a are defined by

$$A_{\mathrm{FB}}^{q\bar{q}} = \frac{3}{4}\frac{\sigma_{\mathrm{F}}}{\sigma} = \frac{3}{4}\frac{\sigma_{\mathrm{F}}}{\sigma_{\mathrm{U}} + \sigma_{\mathrm{L}}} \ , \tag{3}$$

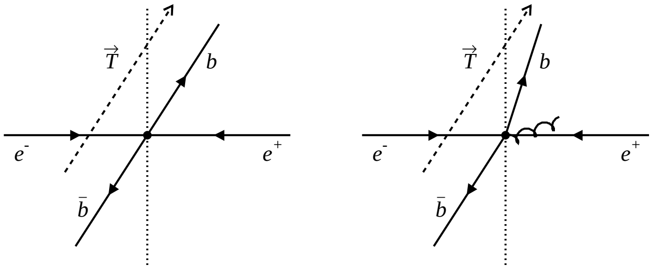
$$a = \frac{\sigma_{\mathrm{U}} - 2\sigma_{\mathrm{L}}}{\sigma_{\mathrm{U}} + 2\sigma_{\mathrm{L}}} \ . \tag{4}$$

In the case of hadronic decays, the direction of the final state fermion is not accessible experimentally and is usually approximated by the thrust axis direction. Its orientation can be defined using different experimental techniques, based on the charge correlation between the quark and its decay products. As already mentioned, QCD corrections affect the forward-backward asymmetry as well as the shape coefficient. In Fig. 1a–d some of the topologies at first order in α_s which influence the angular distribution are shown. The different examples in this figure show cases with a no gluon radiation, b gluon radiation not affecting the hemisphere/charge assignment, c gluon radiation flipping the charge assignment for quark and thrust direction and d flipping the charge assignment only for the quark direction.

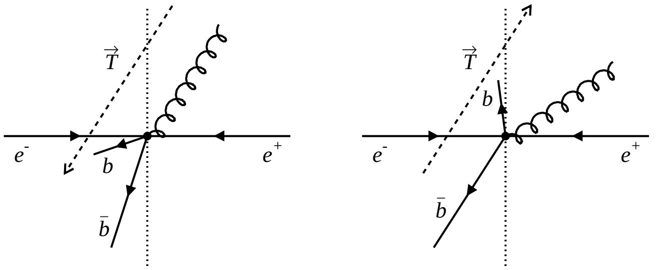
The asymmetry $(A_{\mathrm{FB}}^{q\bar{q}})_0$ determined using the direction of the quark q without gluon radiation, is related to $A_{\mathrm{FB}}^{q\bar{q}}$ by

$$A_{\mathrm{FB}}^{q\bar{q}} = (1 - C_{\mathrm{QCD}})(A_{\mathrm{FB}}^{q\bar{q}})_0 \ , \tag{5}$$

where C_{QCD} is the correction coefficient which accounts for gluon emission, and which can be calculated in perturbative QCD.



(a) No gluon (b) Soft gluon



(c) Hard gluon (d) Thrust forward, quark backward

Fig. 1a–d. Pictorial representation of final state topologies with gluon emission. For a three-parton final state the thrust axis is defined by the direction of the parton with the highest momentum. In this figure the thrust axis is oriented that $\mathbf{T}\cdot\mathbf{p} > 0$, with $\mathbf{p}$ corresponding to the quark direction. **a** The pure electroweak process. **b** "Soft" gluon radiation slightly changes the direction of the quark. The thrust axis is a good estimator of the primary b direction. **c** "Hard" gluon radiation flips the quark in the hemisphere of the anti-quark. No charge information is contained in the hemisphere of the gluon. **d** Another topology caused by hard gluon radiation is represented. The event is classified as "forward" when using the thrust direction even if the quark has flipped into the backward hemisphere

3 Theoretical estimate of the QCD corrections

Estimates of QCD corrections to heavy quark forward-backward asymmetries have been computed at a fixed order in α_s by several authors [2–4]. Most calculations use the direction of the quark to define the axis relative to which the asymmetry is computed. In [3] first order QCD corrections are given where the direction of the thrust is used, calculated using all partons in the final state. These calculations include mass corrections. Since in experimental measurements the quark direction is approximated by the reconstructed thrust axis these results are used as the basis of the proposed corrections. To estimate the size of the higher order effects, the second order QCD corrections given in [4] are also used, although they correspond to massless quarks and use the quark direction as the reference.

The correction to the asymmetry at the scale $\mu^2 = m_{\mathrm{Z}}^2$ is parametrised by

$$\begin{aligned} A_{\mathrm{FB}}^{q\bar{q}} &= (1 - C_{\mathrm{QCD}})\,(A_{\mathrm{FB}}^{q\bar{q}})_0 &\quad (6)\\ &= \left(1 - \frac{\alpha_s\left(m_{\mathrm{Z}}^2\right)}{\pi}c_1 - \left(\frac{\alpha_s\left(m_{\mathrm{Z}}^2\right)}{\pi}\right)^2 c_2\right)(A_{\mathrm{FB}}^{q\bar{q}})_0 \ , \end{aligned}$$

where the parameters c_1 and c_2 give the contributions of first and second order terms, respectively. The method used to obtain the value and uncertainty of these parameters is explained in the following.

- The first order calculation of [3], where the asymmetry is defined from the parton-level thrust direction, provides the value of c_1. For completeness results obtained using the quark direction for the definition of the asymmetry are also quoted.
- For the strong coupling constant a value of $\alpha_s\left(m_{\mathrm{Z}}^2\right) = 0.119 \pm 0.004$ [7] is used.

Table 1. Values of c_1 from [3] for different mass hypotheses and two different definitions of the event axis (quark or thrust). For the second order QCD corrections, c_2, a value corresponding to $m_q = 0$ and the quark direction from [4] is given. All other numbers in this table are estimated as explained in the text

Event axis	mass in GeV/c^2	$b\bar{b}$ events m_b=0	$b\bar{b}$ events m_b=3	$b\bar{b}$ events m_b=4.5	$c\bar{c}$ events m_c=0	$c\bar{c}$ events m_c=0.7	$c\bar{c}$ events m_c=1.5
quark	c_1 [3]	1.	0.86	0.80	1.	0.96	0.93
	c_2 [4]	1.9 ± 0.4	–		4.6 ± 0.4	–	
	$C_{\mathrm{QCD}}^{\mathrm{quark}}$ (%)		3.30 ± 0.37			4.18 ± 0.69	
thrust	c_1 [3]	0.89	0.81	0.77	0.89	0.88	0.86
	c_2		–			–	
	$C_{\mathrm{QCD}}^{\mathrm{part,T}}$ (%)		3.19 ± 0.33			3.92 ± 0.68	

- In the first order calculations the masses of the quarks are taken to be the pole masses ($m_b = 4.5$ GeV/c^2 and $m_c = 1.5$ GeV/c^2). The size of the unknown higher order corrections in the mass dependent terms is estimated by re-evaluating the correction choosing running masses at the m_{Z} scale ($m_b = 3$ GeV/c^2 and $m_c = 0.7$ GeV/c^2). The full difference between the two estimates is taken as a systematic uncertainty. The mass dependence of the Born term is negligible.
- In [4] second order QCD calculations based on the quark direction and without mass corrections are given; these results, updated using $m_t = 175 \pm 6$ GeV/c^2, are used to estimate c_2. The full size of the second order term is also taken as additional systematic uncertainty on the correction; its effect is however much smaller than the impact of the quark mass, or the difference between using the quark direction or the thrust direction to define the event axis, as can be seen from Table 1.
- A special second order QCD process is the splitting of hard gluons to $b\bar{b}$ and $c\bar{c}$ pairs. This effect is not considered in the paper since present analyses account for it by treating this source of heavy quarks as background.

A detailed summary of the different values of c_1 and c_2 [3, 4] is given in Table 1 together with the resulting QCD corrections. Two sets of numbers are given, $C_{\mathrm{QCD}}^{\mathrm{quark}}$, corresponding to the quark direction, and $C_{\mathrm{QCD}}^{\mathrm{part,T}}$, referring to the parton-level thrust direction. The different contributions to the uncertainty on $C_{\mathrm{QCD}}^{\mathrm{quark}}$ and $C_{\mathrm{QCD}}^{\mathrm{part,T}}$ are listed in Table 2. It should be noted that:

- the use of the thrust axis instead of the quark direction slightly reduces the QCD corrections (by 5 to 10%) as illustrated in Fig. 1a–d;
- the QCD corrections decrease when the mass of the quark considered increases as would be expected from Galilei's law of inertia. When the quark direction is used, the estimated QCD correction is reduced by 20% for $m_b = 4.5$ GeV/c^2 compared to the massless case. This mass effect is less pronounced when the thrust axis is used.

The shape coefficient a has been calculated at first order QCD [5]. In general a does not depend heavily on the mass or flavour, but changes significantly when going from the quark to the thrust direction. The actual value of a is 0.91 ± 0.02 when the quark direction is used and 0.94 ± 0.01 when using the thrust. The error originates mainly from the uncertainty in the renormalisation scale in the first order, estimated by varying α_s between 0.10 and 0.14; it also includes the uncertainty in the quark masses. The effect of the shape coefficient on the asymmetry determination is closely related to the technique used to analyse the experimental data, depending in particular on the fitting method, and will be discussed in Sect. 5.

Table 2. Different sources of uncertainty on the QCD corrections, when the quark or the thrust directions are used. The total errors correspond to the values reported in Table 1

Error source	$C_{\mathrm{QCD}}^{\mathrm{quark}}$ (%) $b\bar{b}$	$C_{\mathrm{QCD}}^{\mathrm{quark}}$ (%) $c\bar{c}$	$C_{\mathrm{QCD}}^{\mathrm{part,T}}$ (%) $b\bar{b}$	$C_{\mathrm{QCD}}^{\mathrm{part,T}}$ (%) $c\bar{c}$
Theoretical error on m_b or m_c	0.23	0.11	0.15	0.08
$\alpha_s(m_{\mathrm{Z}}^2)$ (0.119 ± 0.004)	0.12	0.16	0.12	0.16
Higher order corrections	0.27	0.66	0.27	0.66
Total error	0.37	0.69	0.33	0.68

4 Generator studies and hadronisation effects

The results quoted in the previous section are not directly applicable to the measurements, since experimentally the thrust is computed from stable particles observed in the detector and not from partons. The only way to estimate the change in the QCD corrections between the two frameworks is by means of Monte Carlo simulations. In this section, the study is limited to the ideal case where all final particles are detected. In the next section results corresponding to the experimental conditions are given.

The results obtained in Sect. 3 can be compared with different simulations using three definitions of the event direction:

- $C_{\mathrm{MC}}^{\mathrm{quark}}$, calculated using the direction of the quark,

Table 3. Different values of C_{QCD} in % (see (5)) and a (see (2)) for $e^+e^- \to b\bar{b}$ events. The line quoted as JETSET 7.408 is obtained using the JETSET generator with default tuning. All the other lines correspond to results obtained with generators tuned by the LEP experiments. The lines quoted with a † are used in Table 5 to estimate the reference values of the QCD corrections at the level of the hadrons. The last line corresponds to the theoretical estimates presented in Sect. 3. The statistical errors on a obtained with the generators are negligible compared to the theoretical errors and therefore are not quoted

$b\bar{b}$ events	$C_{\mathrm{MC}}^{\mathrm{quark}}$	$C_{\mathrm{MC}}^{\mathrm{part,T}}$	$C_{\mathrm{MC}}^{\mathrm{had,T}}$	$a_{\mathrm{MC}}^{\mathrm{quark}}$	$a_{\mathrm{MC}}^{\mathrm{part,T}}$	$a_{\mathrm{MC}}^{\mathrm{had,T}}$
JETSET 7.408	3.95 ± 0.02	3.17 ± 0.02	2.94 ± 0.02	0.89	0.95	0.96
ALEPH JETSET [10] †	3.84 ± 0.09	3.07 ± 0.04	2.79 ± 0.08	0.90	0.96	0.98
DELPHI JETSET [11] †	4.38 ± 0.03	3.43 ± 0.03	3.06 ± 0.03	0.88	0.95	0.95
L3 JETSET [12] †	3.98 ± 0.06	3.15 ± 0.06	2.89 ± 0.06	0.89	0.95	0.98
OPAL JETSET [13] †	4.02 ± 0.18	3.04 ± 0.19	3.02 ± 0.20	0.89	0.95	0.95
OPAL HERWIG [13]	4.11 ± 0.14	2.86 ± 0.13	2.75 ± 0.14	0.90	0.95	0.99
	$C_{\mathrm{QCD}}^{\mathrm{quark}}$	$C_{\mathrm{QCD}}^{\mathrm{part,T}}$		$a_{\mathrm{QCD}}^{\mathrm{quark}}$	$a_{\mathrm{QCD}}^{\mathrm{part,T}}$	
Theory	3.30 ± 0.37	3.19 ± 0.33		0.91 ± 0.02	0.95 ± 0.01	

Table 4. Different values of C_{QCD} in % (see (5)) and a (see (2)) for $e^+e^- \to c\bar{c}$ events. The line quoted as JETSET 7.408 is obtained using the JETSET generator with default tuning. All the other lines are calculated using generators tuned by the LEP experiments. The lines quoted with a † are used in Table 5 to estimate the reference values $C_{\mathrm{QCD}}^{\mathrm{had,T}}$. The last line corresponds to the theoretical estimates presented in Sect. 3

$c\bar{c}$ events	$C_{\mathrm{MC}}^{\mathrm{quark}}$	$C_{\mathrm{MC}}^{\mathrm{part,T}}$	$C_{\mathrm{MC}}^{\mathrm{had,T}}$	$a_{\mathrm{MC}}^{\mathrm{quark}}$	$a_{\mathrm{MC}}^{\mathrm{part,T}}$	$a_{\mathrm{MC}}^{\mathrm{had,T}}$
JETSET 7.408	5.49 ± 0.04	3.95 ± 0.04	3.51 ± 0.04	0.85	0.95	0.95
DELPHI JETSET [11]†	5.74 ± 0.03	4.12 ± 0.03	3.65 ± 0.03	0.85	0.95	0.95
OPAL JETSET [13]†	4.95 ± 0.18	4.03 ± 0.18	3.80 ± 0.19	0.86	0.96	0.96
OPAL HERWIG [13]	4.75 ± 0.17	3.18 ± 0.16	3.26 ± 0.17	0.87	0.97	0.97
	$C_{\mathrm{QCD}}^{\mathrm{quark}}$	$C_{\mathrm{QCD}}^{\mathrm{part,T}}$		$a_{\mathrm{QCD}}^{\mathrm{quark}}$	$a_{\mathrm{QCD}}^{\mathrm{part,T}}$	
Theory	4.18 ± 0.69	3.92 ± 0.68		0.91 ± 0.02	0.95 ± 0.01	

- $C_{\mathrm{MC}}^{\mathrm{part,T}}$, calculated using the direction of the parton-level thrust axis oriented according to the direction of the quark,
- $C_{\mathrm{MC}}^{\mathrm{had,T}}$, calculated using the direction of the thrust axis from all stable particles (including neutrinos), oriented according to the direction of the weakly decaying hadron containing the quark.

The values of $C_{\mathrm{QCD}}^{\mathrm{quark}}$ and $C_{\mathrm{QCD}}^{\mathrm{part,T}}$ obtained with analytical calculations have been presented in the previous section and can be directly compared to the results obtained with Monte Carlo simulations. In Tables 3 and 4 the results of the analytical calculations for $b\bar{b}$ and $c\bar{c}$ events, respectively, are compared with the results obtained using standard JETSET 7.4 Monte Carlo [8] or different versions of tuned JETSET and HERWIG [13] by ALEPH, DELPHI, L3 and OPAL. The tuned generators include an up-to-date description of production and decay of b and c hadrons together with an improved description of global observables. Also shown are results for the shape parameter a.

In general for the corrections to the asymmetries the agreement between analytical calculations and generators is poor when the quark direction is used (more than 30% difference in one case) while there is a much better agreement using the thrust axis (below 10% discrepancy). Qualitatively this pattern is expected since in a parton shower simulation many more gluons are radiated than in a fixed $O(\alpha_s^2)$ calculation. For $e^+e^- \to b\bar{b}$ the shape parameter a is well reproduced by the simulation. This is not the case for $e^+e^- \to c\bar{c}$ at the quark level. The relatively good agreement between Monte Carlo models and analytical calculations when the thrust direction is used, encourages the use of the simulation to estimate the bias in the QCD corrections induced by experimental techniques.

Table 5. Estimated value of $C_{\mathrm{QCD}}^{\mathrm{had,T}}$. The average value of $\left\langle C_{\mathrm{MC}}^{\mathrm{had,T}} - C_{\mathrm{MC}}^{\mathrm{part,T}} \right\rangle$ is obtained using the results quoted with a † in Tables 3 and 4

	$b\bar{b}$ events	$c\bar{c}$ events
$\left\langle C_{\mathrm{MC}}^{\mathrm{had,T}} - C_{\mathrm{MC}}^{\mathrm{part,T}} \right\rangle$ (%)	-0.23	-0.35
$C_{\mathrm{QCD}}^{\mathrm{had,T}}$ (%)	2.96 ± 0.40	3.57 ± 0.76

The estimates of $C_{\mathrm{MC}}^{\mathrm{had,T}}$ are significantly smaller than $C_{\mathrm{MC}}^{\mathrm{part,T}}$. The source of the difference is the hadronisation itself, the decays of b and c hadrons having a smaller effect. It is interesting to note that the hadronisation decreases the correction. A possible explanation within JETSET is the reconnection of the quark after showering with quarks and gluon that are closer to the original quark direction. No analytical calculation exist for this non-perturbative effect.

Tables 3 and 4 show that the predictions of the various versions of generators have some spread. Even the same generator with different tunings yields different results for $C_{\text{MC}}^{\text{quark}}$ and $C_{\text{MC}}^{\text{part,T}}$ (up to 13% discrepancy). One of the possible sources of difference between different tunings is the cut-off chosen for the energy of gluons emitted in the parton shower. The spread is slightly smaller at the hadron level, $C_{\text{MC}}^{\text{had,T}}$, where the simulation programs are tuned to the experimental data.

The final QCD corrections, $C_{\text{QCD}}^{\text{had,T}}$, are obtained from the analytical calculations on the parton level thrust described in Sect. 3 and the Monte Carlo prediction for the hadronisation effects as

$$C_{\text{QCD}}^{\text{had,T}} = C_{\text{QCD}}^{\text{part,T}} + \left\langle C_{\text{MC}}^{\text{had,T}} - C_{\text{MC}}^{\text{part,T}} \right\rangle \ ,$$

where $\left\langle C_{\text{MC}}^{\text{had,T}} - C_{\text{MC}}^{\text{part,T}} \right\rangle$ is the average of the differences between the QCD corrections at hadron and parton level based on the JETSET estimates quoted in Tables 3 and 4.

The uncertainties for the parton level corrections are those given in Table 2. In addition the full difference between the parton level and the hadron level correction as predicted by the Monte Carlo is taken as the systematic error. The values for the final correction, including its error, are listed in Table 5.

5 The experimental bias

Experimental analyses have different sensitivity to QCD effects, therefore a bias factor, s_q, has to be introduced to scale the QCD corrections previously evaluated with the experimental sensitivity:

$$A_{\text{FB}}^{q\bar{q}} = \left(1 - s_q \times C_{\text{QCD}}^{\text{had,T}}\right) (A_{\text{FB}}^{q\bar{q}})_0 = (1 - C_q)\,(A_{\text{FB}}^{q\bar{q}})_0 \quad (7)$$

Two different types of asymmetry analyses exist at LEP. Either, one of the hadrons containing the heavy quark or antiquark is tagged, e.g. by a reconstructed D^* or a high $p_\perp$ lepton, or the hemisphere of the quark is tagged using all the tracks in the event, typically using a jetcharge algorithm.

For tagged hadron analyses three main effects have been identified which cause s_q to be different from one:

- the analysis cuts, which can introduce a bias with respect to events with hard gluon radiation;
- the polar angle distribution assumed for the selected events, and more generally the fitting method;
- the event axis reconstruction.

Analysis cuts: The analysis cuts can directly influence the size of the QCD corrections, if a selection criterion is sensitive to the strength of the gluon emission, and thereby to the event topology. For example selecting heavy hadrons with large momentum reduces the phase space available for gluon radiation, and therefore the sensitivity of the measurement to QCD effects. This is illustrated in Fig. 2 for the case of an asymmetry measurement based on leptons. With increasing lepton momentum the size of the QCD corrections decreases significantly. In $A_{\text{FB}}^{b\bar{b}}$ or $A_{\text{FB}}^{c\bar{c}}$ analyses based on leptons, a typical cut of 3 GeV/c on the lepton momentum rejects a large fraction of events with hard gluon emission. Similar changes in the sensitivity to QCD effects are also observed if events have different statistical weight in the analysis, depending on their properties. For example in b asymmetry analyses in which a simultaneous fit to the lepton momentum and transverse momentum spectra is performed, the statistical weight of the hardest part of the lepton sample is increased, reducing the sensitivity of the measurement to gluon emission.

Fitting method: Usually asymmetries are extracted via a fit to the event polar angle distribution, based on (2), with the shape parameter a set to one. Fitting provides a natural way to treat the angular acceptance and efficiency of the detectors. The choice of a influences slightly the measured asymmetries, and the effect is absorbed in the QCD corrections. It has been estimated with the simulation that the QCD correction is reduced by about 0.004 for a simple χ^2 fit on a $b\bar{b}$ sample with no angular acceptance cut when a is set to one instead of its true value.

Event axis reconstruction: The reconstructed thrust axis differs from the thrust axis computed from all final-state particles because of unseen particles, and because of the acceptance and resolution of the detectors in energy and angle. For example, the simulation predicts that the change in the QCD correction to $A_{\text{FB}}^{b\bar{b}}$ due to the undetected neutrinos is about 0.002.

For the second type of analyses, most of the effects discussed for the tagged hadron techniques still apply. However in the case of jetcharge based analyses, since gluon emission reduces the average charge separation between the hemispheres, which is determined from the data, a large part of the QCD corrections is implicitly accounted for. In these analyses the basic effect of gluon emission is not a shift in the measured asymmetry, but a reduction in the statistical power of the method. Due to the strong interconnection between detector and QCD effects, a global correction is estimated usually by simulation. However it is possible to extract the contribution of the QCD corrections a posteriori.

In Table 6 typical ranges for the experimental bias are shown, as determined for the currently available asymmetry measurements at LEP. On average the QCD corrections are significantly reduced by experimental effects.

6 Conclusions

In this paper, QCD corrections to the heavy flavour forward-backward asymmetries are estimated using previously published analytical calculation [3,4] together with an evaluation of the effect of the hadronisation with Monte

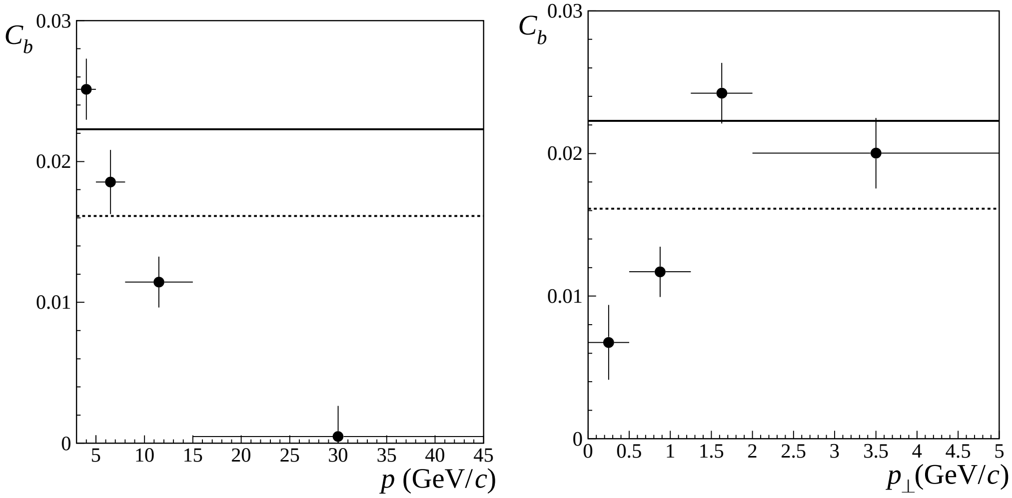


Fig. 2. QCD corrections factors C_b estimated using ALEPH Monte Carlo for the $A_{\mathrm{FB}}^{b\bar{b}}$ analysis using leptons. The QCD corrections are presented as a function of the momentum (p) and transverse momentum ($p_\perp$) of the lepton candidates. All the candidates ($b \to \ell$, $b \to c \to \ell$, ...) are used here. The dashed line corresponds to the average correction over all the kinematic range (with only the identification cut of $p > 3$ GeV/c applied), while the solid line corresponds to the average correction for the candidates used in the analysis ($p_\perp > 1.25$ GeV/c). The trend as a function of the momentum comes from the anti-correlation between the QCD corrections and the b hadron energy, as explained in the text. The trend as a function of the transverse momentum is due to the interplay between the gluon radiation and the performance of the jet algorithm. In events with hard gluon emission higher values of the reconstructed $p_\perp$ are occasionally obtained

Table 6. Values of the bias, s_q, to the QCD corrections used in [1] for the different LEP measurements. The corresponding values of the QCD corrections, C_q, are also given. Only the highest and lowest values of s_q and C_q, obtained among the four LEP experiments, are quoted. For the C_q the first error quoted is correlated between the different measurements and comes from the error on $C_{\mathrm{QCD}}^{\mathrm{had,T}}$, the second error is dominated by the statistical precision on the evaluation of the bias

		Lepton analysis	D analysis	Jet Charge
s_b	min	0.52 ± 0.06	0.29 ± 0.13	0.24 ± 0.46
	max	0.74 ± 0.07	0.46 ± 0.14	0.36 ± 0.32
C_b (%)	min	$1.54 \pm 0.21 \pm 0.18$	$0.86 \pm 0.12 \pm 0.38$	$0.71 \pm 0.10 \pm 1.36$
	max	$2.19 \pm 0.30 \pm 0.21$	$1.36 \pm 0.18 \pm 0.41$	$1.07 \pm 0.14 \pm 0.95$
s_c	min	0.19 ± 0.11	-0.06 ± 0.09	
	max	0.37 ± 0.08	0.44 ± 0.13	
C_c (%)	min	$0.68 \pm 0.14 \pm 0.39$	$-0.21 \pm 0.05 \pm 0.33$	
	max	$1.32 \pm 0.28 \pm 0.29$	$1.57 \pm 0.33 \pm 0.46$	

Carlo models. The correction at the hadron level, $C_{\mathrm{QCD}}^{\mathrm{had,T}}$, is (2.96 ± 0.40)% for the b asymmetry and (3.57 ± 0.76)% for the c asymmetry. For the b asymmetry this correction is larger than the total error on the current LEP average. It has been shown that the experimental analysis methods considerably bias these numbers and can substantially reduce the correction itself.

When averaging several experimental results it should be taken into account that systematic errors on $C_{\mathrm{QCD}}^{\mathrm{had,T}}$ are correlated between the different measurements. The errors on the bias, s_q, quoted in Table 6 are dominated by Monte Carlo statistics and are uncorrelated among the different measurements. As the error on $C_{\mathrm{QCD}}^{\mathrm{had,T}}$ is scaled by the experimental bias s_q, only a fraction of the correlated error will propagate through to the average of several measurements. The criteria presented in this paper were used for the LEP averages of the heavy flavour forward–backward asymmetries presented at the 1997 Summer Conferences [1]. The total errors on the b and c quark asymmetries were 0.0024 and 0.0048 respectively. The contribution of the QCD corrections to these errors was about 0.0003.

The size of the QCD corrections and of the experimental biases indicate clearly that knowledge of both is mandatory in order to extract $\sin^2\theta_{\mathrm{W,eff}}^{\mathrm{lept}}$ from the heavy quark asymmetry measurements. The large spread of the bias for different analyses requires that it is evaluated individually for each measurement.

Acknowledgements. We would like to thank G. Altarelli, W. Bernreuther, A. Djouadi, B. Lampe and P. Zerwas for their useful comments. We are greatly indebted to our colleagues from the four LEP experiments for their support.

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